More than Metrology: Mathematics Education in an Old Babylonian Scribal School

Eleanor Robson, Oxford

1. Introduction

1.1 Approaches to OB mathematics and scribal education

For most of the twentieth century the study of Old Babylonian (OB) mathematics quite rightly focussed on the recovery of knowledge: what was known, and where and when. The last decade has seen a move towards conceptual history: how mathematical language reflected the thought processes behind the techniques. Nevertheless, the corpus of Old Babylonian mathematics was treated, more or less, as a closed set of disembodied texts: there were few attempts to publish new sources, or to acknowledge that they were recorded on physical objects which could be located in time and space and fruitfully related to other archaeological artefacts. To be fair, this was in large part due to the academic backgrounds of the small number of core researchers concerned, who were almost exclusively trained in mathematics or the history of science or ideas, and inevitably lacked the technical skills involved in the primary publication of cuneiform tablets or the reconstruction of the archaeological record. Conversely, there was a conspicuous lack of cuneiformists willing to do so. It appeared that the great pioneers of cuneiform mathematical studies - Neugebauer, Sachs, Bruins, and Thureau-Dangin - had done it all, and there was little left to do but reanalyse their data. The contextual evidence for the material they had published was at best meagre and more commonly nonexistent, while the mathematical tablets coming out of more recent excavations were invariably further exemplars of the multiplication tables and metrological lists that had been so thoroughly classified by Neugebauer in the 30s and 40s.

Since the mid-1990s, however, there has been an increasing interest in the material culture of scribal schooling, and a growing realisation that there is a wealth of archaeological and artefactual data which can be used to counterbalance the traditional sources of evidence, the Sumerian literary narratives about school. This move has gone hand in hand with an increasingly sophisticated approach to textual evidence, which acknowledges that authorial intention was often complex and that literary text in particular cannot be used straightforwardly as a historical source.

This study is situated firmly within that research tradition. It takes as its starting point one single architectural unit and the objects found within it to reconstruct the role of metrology, arithmetic, and mathematics within the curriculum of an

individual school. Its aim is not to produce a generalised scholastic framework for Old Babylonian mathematics but rather, through comparison with pieces of evidence from other contexts, to stress the variety of approaches to mathematics education that existed in the early second millennium BC. Just as modern scholarship is conducted by individuals who are constrained by their environment and education while free to make personal choices about the direction and character of their work so, we shall see, was ancient education imparted by people with similar freedoms and constraints.

1.2 The history of House F

House F was excavated in the first months of 1952 by a team of archaeologists from the universities of Chicago and Pennsylvania. It was their third field season in the ancient southern Iraqi city of Nippur and one of their express aims was to find large numbers of cuneiform tablets.¹ For this reason they had chosen two sites on the mound known as Tablet Hill, because of the large number of tablets that had been found there in the late nineteenth century. Those previous digs, though, when the development of recorded stratigraphic archaeology was still in its infancy, had necessarily been little more than hunts for artefacts. The new generation of archaeologists labelled their excavation areas TA and TB (Figure 1), deliberately siting TB right next to the pits left by their nineteenth century predecessors.² Now, however, they made detailed archaeological records of finds and findspots as a matter of course, so that when they hit upon the large cache of tablets they had been hoping for, the architectural context, stratigraphic location and physical description of every one of them was noted.³ The tablets, over 1400 of them, were in an unremarkable looking house in the corner of Area TA, one of eight mud-brick dwellings packed into the 20×40 m rectangle. Other houses in TA and TB had yielded tablets, but in handfuls or dozens, not in the thousands. The House F tablets were not stored in jars or discarded on the street as some of the others had been, but were part of the very fabric of the house itself, built into the floors and walls and furniture (Figure 2). It quickly became apparent that the tablets were not a normal household archive of documents relating to property ownership, debt, and business matters but comprised in the most part Sumerian literary compositions and pieces of lexical lists, in numbers that had never before been recovered from a controlled excavation.

The excavators of House F had found a school. While the huge number of school tablets were not enough to confirm this at the time, having been found in secondary context used as construction rubble, the presence of large quantities of unused tablet clay and facilities for soaking and reusing tablets has since been attested in other schooling environments and leaves little room for doubt. The schooling took place, it appears, in the courtyards, loci 192 and 205, where benches and three recycling bins were found. Three small rooms to the northwest, 184, 189, and 191, seem to have been private quarters (a bread oven was discovered in 191, domestic pottery in

¹ McCown and HAINES (1967), p. viii.

 $^{^2}$ GIBSON *et al.* (2001) give a thorough overview of the excavations at Nippur and their results.

³ Nevertheless, the 1950s field records still present major problems for researchers: see ZETTLER (1996), pp. 88–89.

184 and 205, and decorative plaques in 191 and 205), while the partially excavated 203 must have served as the entrance hall. The tablets, it seems, were laid down shortly after 1740 BC, the tenth regnal year of Samsu-iluna, king of Babylon and son and successor of Hammurabi.⁴ Mudbrick structures like House F needed to be rebuilt or extensively renovated every 25 years or so⁵ and indeed House F appears to have undergone three or four such remodellings over the course of its life, from the late nineteenth century to about 1721 BC. The use of tablets as building material seems to mark the end of the house's life as a school: when it was later reoccupied the new inhabitants appear to have been engaged in other activities.



Figure 1: Topographic map of Nippur. Area TA is Sigure 2: Plan of House F, south of Inana's temple (GIBSON et al. (2001), fig. 1). level 10 (After STONE (1987), pls. 17–19).

After excavation, the finds were distributed between the Oriental Institute, Chicago and the University Museum, Philadelphia. The Iraq Museum in Baghdad also took a share, while sending portions of its allocation to Chicago and Philadelphia on long-term loan for publication purposes.⁶ The Chicago loan was returned in the 1980s. The excavation report on TA and TB came out fifteen years after the dig⁷ and was

 $^{^4}$ The Middle Chronology, which puts Hammurabi's reign at 1792–50 BC, is followed throughout this paper.

⁵ GASCHE and DEKIERE (1991).

⁶ Chicago: 159 tablets plus 347 on long-term loan (now returned; plastercasts of these fragments are kept in Chicago); Philadelphia: 200 tablets plus 533 on long-term loan (still in Philadelphia; HEIMERDINGER (1979)); Baghdad: 441 tablets (some plastercasts of which are in Chicago and Philadelphia) plus the 347 returned from Chicago. Total 1680 3N-T tablets from TA, of which 1425 are from House F, 209 from other houses, and 46 from undetermined locations in TA.

⁷ McCown and HAINES (1967).

later reanalysed in the light of information from about 100 household archival records from the two sites.⁸ Neither work treated in any detail the school tablets from House F or the rest of the site. Meanwhile, many of those tablets, identifiable by the siglum 3N-T, were making their way into critical editions of Sumerian literary and lexical works, sometimes contributing as much as 25% of the sources. Only the fragments loaned to Philadelphia were systematically published,⁹ but even this work consisted solely of cuneiform copies with no textual commentary, edition, or discussion of the archaeological context. As the tablets were published, it became clear that fragments could be joined to form larger pieces of tablets, but that often these joins had to be made virtually, across the three collections. Nevertheless, the preservation in Philadelphia of the original field notebooks containing a complete record of the epigraphic finds as they were excavated, as well as a tablet catalogue drawn up in the 1970s, has made it possible to attempt a reconstruction of the original tablet assemblage. As Baghdad becomes more accessible to the international community, it is now also feasible to check those records against the tablets in the Iraq Museum. As this process has only just begun, there are still some gaps and inconsistencies in the data, which should therefore not be taken as completely accurate. Nevertheless, it is already possible to make some interesting observations and draw some preliminary conclusions about the functioning of the House F school in the mid-eighteenth century BC.

Subject	Number of tablet pieces	Percentage of total
Mathematics	127	8.9
Other elementary subjects	591	41.5
Sumerian literature	591	41.5
Non-school	28	1.9
Unidentified	88	6.1
Total	1425	

Figure 3: Subject matter of the tablet pieces in House F.

In total 1425 tablets were recovered from Level 10 of House F (Figure 3), some 9 percent of which are mathematical. All but four of these belong to the tablet typology of elementary schooling, which account altogether for 50 percent of the tablets found in the house. The position, content, style, and purpose of mathematical instruction within the House F elementary curriculum will be discussed in §§2–3 below. The bulk of the remaining tablets bear extracts from Sumerian literary compositions (42 percent), some of which may help to shed light on post-elementary mathematical training. This is the topic of §4. The final 8 percent, household documents and hitherto unidentified fragments, will not come into the discussion.

⁸ STONE (1987).

⁹ Heimerdinger (1979).

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1.3 Comparative data

Several other assemblages of Old Babylonian school tablets have been identified, all much smaller than the House F corpus. The better published of them include:

- The 25 elementary school tablets, including 7 mathematical ones, from a small room in Sîn-kāšid's palace in Uruk, c.1860 BC,¹⁰ 120 years before House F.
- The 380-odd identified tablets of a reported 2000 excavated from 'No. 1 Broad Street' in Ur, including about 60 mathematical tablets.¹¹ As in House F, the tablets had been re-used as fill, but it is not clear whether the house itself functioned as a school at any time. The tablets belong to two separate lots: one, almost exclusively of school documents, found in one part of the house, and another, predominantly administrative records covering the period 2020–1787 BC, mixed with further school tablets. The second lot of school tablets, we might therefore guess, dates to about 1790 BC.¹²
- The 180 school tablets, including nine mathematical ones, from the so-called *Scherbenloch* or sherd-pit near the temple complex in Uruk.¹³ Although their original context has been lost, they were found with a coherent group of letters and business documents dating to the 1780s (Rim-Sin 31–42), some forty years before House F.
- The 59 school tablets, of which seven are mathematical, from the 'scholar's library' in Me-Turan.¹⁴ This private house in a provincial town in northern Eshnuna also yielded 22 magico-liturgical tablets and 90 household business documents and letters. It appears to have been destroyed in c.1760 BC, about 20 years before House F.
- The 46 school tablets from 'No. 7 Quiet Street' in Ur,¹⁵ four of which are mathematical. The house burned down in or after 1740 BC (Samsu-iluna's 10th regnal year), making it House F's exact contemporary.
- The 68 elementary school tablets, eight of which are mathematical, found in and around a recycling bin in the main courtyard of a substantial house owned by a family of *gala-mah* priests in Sippir Amnānum.¹⁶ The house was destroyed by fire in 1629 BC (Ammi-sadūqa's 18th regnal year), but the school tablets were deposited in an earlier occupation phase, around 1640 BC, a hundred years after the floruit of House F.

There are also more sparsely documented finds of Old Babylonian school tablets, from Nippur, Larsa, Isin, Susa, and towns in the Diyala Valley.

House F thus falls in the middle of the chronological and geographical spread of these school corpora. Mathematics is present in all of them, in proportions ranging from 5 percent in the *Scherbenloch* to 16 percent in No. 1 Broad Street; the 9 percent proportion of mathematics in House F can thus be seen as relatively

¹⁰ CAVIGNEAUX (1982).

¹¹ Charpin (1986), pp. 451–452; Robson (1999), pp. 245–272.

¹² The mathematics from Ur has recently been the subject of a study by FRIBERG (2000).

¹³ CAVIGNEAUX (1996); VELDHUIS (1997–1998).

¹⁴ CAVIGNEAUX (1999).

¹⁵ Charpin (1986).

¹⁶ TANRET (1982); GASCHE (1989); TANRET (2002).

normal.¹⁷ However, the contents and format of the mathematical tablets in the seven different assemblages, as far as they can be identified, vary quite remarkably.

2. Metrology in the elementary curriculum

2.1 The elementary curriculum in House F

Five types of tablet were used for elementary schooling in Nippur. This classification was first used to describe lexical lists — standardised lists of signs and words — as follows:

Type I refers to generally large tablets [...], with a full lexical list and a substantial part thereof and nothing else.

Type II [...] tablets contain divergent material on each of [their] two sides. To the left of the flat side (II/1) there is a carefully written lexical passage extracted from a fuller list, apparently the work of an instructor, while to the right the passage is copied by a student. On the convex side (II/2) of a Type II tablet, there is a multicolumn excerpt from a longer list.

Type III tablets [...] contain just one column with material extracted from a longer list.

Type IV are plano-convex (flat on one side and convex on the other) round (lenticular) tablets [...]. On the flat side, they have two to four lines written by the instructor and copied underneath by the student. On some of them, the convex side gives the reading of the signs in syllabograms and/or their Akkadian translation.¹⁸

As Niek Veldhuis has shown, however, this tablet typology applies equally to all elementary school exercises including mathematical ones.¹⁹ The fifth tablet type, the prisms, contain similar material to Type I tablets; we shall refer to these as Type P. It happens that no mathematics has survived on Type IV or Type P tablets from House F;²⁰ we will thus be dealing with just three types: the small Type IIIs and the larger Type I and IIs, of which it will be important to distinguish obverse (Type II/1) and reverse (II/2).

Because the Type II tablets contain different compositions on the obverse and reverse they can be used to reconstruct the curricular sequence of elementary education in Nippur (see Figure 3). Veldhuis correlated the contents of obverse and reverse on some 1500 Type II tablets from Nippur, working from the hypothesis that they had been written by the same student, who reviewed on the reverse an earlier part of the same composition he was learning on the obverse, or long sections from

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¹⁷ The school finds from Sîn-kāšid's palace and the gala-mahs' house are solely elementary exercises; mathematical tablets account for 28 percent and 12 percent respectively of those find-groups compared to 18 percent of the House F elementary tablets.

¹⁸ CIVIL (1995), p. 2308.

¹⁹ VELDHUIS (1997), pp. 28–39.

²⁰ But it is known from other sites: for instance MDP 27: 61, a Type IV tablet from Susa with a 3 times multiplication table (VAN DER MEER (1935), p. 61) and AO 8865, a six-sided Type P prism from Larsa, dated 1749 BC, bearing standard lists of squares, inverse squares and inverse cubes (NEUGEBAUER (1935–1937), I, pp. 71–75). See also §4.4 below.

_	Phase/Composition	Educational function	No. of tablets in House F ²¹
	First Phase: writing techniques		146
0	Exercises in sign forms	Writing single and combined cuneiform wedges	1
1	Syllable Alphabet B	The proper formation of simple cuneiform signs	70
2	Lists of personal names (^d inana- téš)	The combination of signs into meaningful sense units	82
	Second Phase: thematic vocabulary acquisition: realia		98
3	Division 1: List of trees and wooden objects		28
4	Division 2: List of reeds, vessels, leather, and metal objects		20
5	Division 3: List of animals and meats		19
6	Division 4: List of stones, plants, fish, birds, and garments		25
7	Division 5: List of geographical names and terms, and stars		6
8	Division 6: List of foodstuffs		7
	Third Phase: advanced lists		207
10	Nigga }		16
10	Proto-Kagal } order uncertain	Ordered by key signs	11
12	Proto-Izi }	Thematic vocabulary	30 22
12		acquisition: titles and professions	22
13	Proto-Ea	Sumerian readings of signs	17
14	Metrological lists and tables	Weights and measures	15
15	Multiplication and reciprocal tables	Number facts	93
16	Proto-Diri	The readings of compound signs	16
	Fourth Phase: introductory Sumerian		107
17	Model contracts	Simple Sumerian prose	54
18	Proverbs	Sumerian literary language	54

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Figure 4: The elementary curriculum in House F.²²

 $^{^{21}~}$ Of all tablet types, not only Type II. The numbers in the column are not commensurate because of the co-occurrence of different compositions on the Type II tablets.

²² This table excludes apparently extra-curricular compositions which are attested only once or twice in the house, such as OB Lu A and Proto-Aa, and which cannot easily be assigned a position in the curriculum; see too VELDHUIS (1997), p. 59.

one he had completed earlier.²³ His results were impressively consistent, and enabled him to assign about twenty different compositions to four phases of the elementary curriculum: writing techniques, thematic noun lists, advanced lists, and introductory Sumerian. He discussed the educational function of each phase in turn, showing a steady progression from first exposure to the physical form of cuneiform signs, the construction of whole words, and the exploration of the complexities of cuneiform writing, to the use of whole sentences of grammatically correct Sumerian. All phases involved the rote memorisation of set texts, mainly of the sort traditionally characterised in the field as 'lexical texts', namely, lists of cuneiform signs or Sumerian words. But the curriculum also included model legal documents, Sumerian proverbs, and—most importantly for our purposes—long sequences of multiplication tables and lists of metrological units. It is impossible, though, on present evidence, to estimate how long the student(s) had been at school, or how old they were, at this or any other point in their educational careers.

Using the same methodology on the 250 or so Type II tablets from House F yields a similarly consistent picture (Figure 4). It differs from Veldhuis' general conclusions only in the omission of the syllable list *tu-ta-ti* from the first phase and in the ordering of the third phase, where mathematical matters are addressed.²⁴ Weights and measures were learned systematically, by means of a standard series, towards the end of the third phase of the House F curriculum (see §2.3). Multiplication and division facts were memorised immediately afterwards (see §3). However, metrological matters were first addressed within the second phase, as sequences within the thematic noun list (see §2.2) and later contextualised in the model legal contracts of phase four (§2.4).

Looking at the numbers of tablets attested, it is striking that the series of divisions and multiplications is one of the most frequently occurring compositions, while the metrological sequence is among the least represented. Why this might be, if it is not simply an accident of preservation, cannot for the moment be determined.

2.2 Metrological sequences in the thematic noun lists

The students' first exposure to metrological notation was in the second phase of elementary education, as sub-sequences within the six-part thematic noun list. In the first division, the list of trees and wooden objects, students met the main capacity measures in descending order. Larger capacity measures (c.1,500–18,000 litres) were contextualised as standard sizes of boat within the 60-line section on boats (lines 261–320). Smaller units (c.0.17–60 litres) were treated later on, in a section of their own:²⁵

1.279	ĝiš-ma ₂ -60-gur	Boat of 60 gur capacity (1 gur = $c.300$ litres)
1.280	ĝiš-ma ₂ -50-gur	Boat of 50 gur capacity
1.281	ĝiš-ma ₂ -40-gur	Boat of 40 gur capacity

²³ VELDHUIS (1997), pp. 40–63.

²⁴ ROBSON (forthcoming).

²⁵ The list of trees and wooden objects also includes an obscure section, apparently on scribal apparatus (lines 142–159; VELDHUIS (1997), pp. 86–88), which has attracted sporadic attention in the mathematical literature (e.g., WASCHKIES (1989), p. 87). It includes $\tilde{g}i$ - sa_4 -lum "measuring stick" (line 142) and $\tilde{g}i$ - $surum_x$ -ma perhaps "accounting board" (line 150) but many of the other entries remain unexplained.

1.282	ĝiš-ma ₂ -30-gur	Boat of 30 gur capacity
1.283	ĝiš-ma ₂ -20-gur	Boat of 20 gur capacity
1.284	ĝiš-ma ₂ -15-gur	Boat of 15 gur capacity
1.285	ĝiš-ma ₂ -10-gur	Boat of 10 gur capacity
1.286	ĝiš-ma ₂ -5-gur	Boat of 5 gur capacity
1.287	ĝiš-ma ₂ -tur	Small boat
(After VEL	DUIS (1997), p. 157)	
1.515	ĝiš−lid₂-ga	Measuring vat
1.516	ĝiš-ba-ri ₂ -ga	Measuring vat of 60 sila capacity ($1 \text{ sila} = c.1 \text{ litre}$)
1.517	ĝiš-ba-an	Measuring vat of 10 sila capacity
1.518	ĝiš-ba-an-5-sila₃	Measuring vat of 5 sila capacity
1.519	ĝiš-niĝ ₂ -2-sila ₃	Measuring vat of 2 sila capacity
1.520	ĝiš-1-sila₃	Measuring vat of 1 sila capacity
1.521	ĝiš-1/2-sila ₃	Measuring vat of 1/2 sila capacity
1.522	ĝiš-1/3-sila ₃	Measuring vat of 1/3 sila capacity
1.523	ĝiš-2/3-sila ₃	Measuring vat of 2/3 sila capacity
1.524	ĝiš-10-gin ₂	Measuring vat of 0;10 sila capacity
1.525	ĝiš-5-gin ₂	Measuring vat of 0;05 sila capacity
1.526	ĝiš-3-gin ₂	Measuring vat of 0;03 sila capacity
1.527	ĝiš-2-gin ₂	Measuring vat of 0;02 sila capacity
1.528	ĝiš-1-gin ₂	Measuring vat of 0;01 sila capacity
(After VEL	DHUIS (1997), p. 163)	

Weights were treated very briefly, within a five-line section on weighing equipment, in the list of trees and wooden objects, but were covered more exhaustively, as stone weights, as a section in the list of stones (division four of the thematic noun list).

1.436	ĝiš-rin ₂	Balance
1.437	ĝiš-rin ₂ -lib-lib-bi	Balance arm [?] (Cf. CAVIGNEAUX (1992))
1.438	ĝiš-rin ₂ -1-gu ₂ -un	Balance for 1 talent (c.30 kg)
1.439	ĝiš-rin ₂ -ma-na	Balance for 1 mina (c.0.5 kg)
1.440	ĝiš-e ₂ -rin ₂	Balance box
1.441	ĝiš-dilim ₂ -rin ₂	Balance pan
(After VEI	LDUIS (1997), p. 161)	

4.178	na_4-1-gu_2	Weight of 1 talent
4.179	na ₄ -50-ma-na	Weight of 50 minas
4.180	na ₄ -40-ma-na	Weight of 40 minas
4.181	na ₄ -30-ma-na	Weight of 30 minas
4.182	na ₄ -20-ma-na	Weight of 20 minas
4.183	na ₄ -15-ma-na	Weight of 15 minas
4.184	na ₄ -10-ma-na	Weight of 10 minas
4.183	na ₄ -5-ma-na	Weight of 5 minas
4.182	na ₄ -3-ma-na	Weight of 3 minas
4.187	na ₄ -2-ma-na	Weight of 2 minas
4.188	na ₄ -1-ma-na	Weight of 1 mina
4.189	na ₄ -2/3-ma-na	Weight of 2/3 mina
4.190	na ₄ -1/2-ma-na	Weight of 1/2 mina
4.191	na ₄ -1/3-ma-na	Weight of 1/3 mina
4.192	na ₄ -10-gin ₂	Weight of 10 shekels
4.193	na ₄ -5-gin ₂	Weight of 5 shekels
4.194	na ₄ -3-gin ₂	Weight of 3 shekels
4.195	na ₄ -2-gin ₂	Weight of 2 shekels

4.196	na ₄ -1-gin ₂	Weight of 1 shekel (c.8 g)
4.197	$na_4-2/3$ -gin ₂	Weight of 2/3 shekel
4.198	$na_4-1/2$ -gin ₂	Weight of 1/2 shekel
4.199	$na_4-1/3-gin_2$	Weight of 1/3 shekel
4.200	na ₄ -igi-4-g̃al ₂	Weight of a quarter (shekel)
4.201	na ₄ -igi-5-g̃al ₂	Weight of a fifth (shekel)
4.202	na ₄ -22 1/2-še	Weight of 22 1/2 grains
4.203	na ₄ -20-še	Weight of 20 grains
4.204	na ₄ -15-še	Weight of 15 grains
4.205	na ₄ -10-še	Weight of 10 grains
4.206	na ₄ -5-še	Weight of 5 grains
(After LAN	DSBERGER et al. (1970),	pp. 60–61)

A very few length measures were listed in a section on reed measuring rods in division two, but in general length and area metrology was not covered, presumably because little of it could be related to the sizes of material objects.

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2.112	gi-1-ninda	Reed of 1 rod length (c.6 m)	
2.113	gi-1-kuš ₃	Reed of 1 cubit length (c.0.5 m)	
2.114	gi-1/2-kuš ₃	Reed of 1/2 cubit length	
2.115	gi-1/3-kuš ₃	Reed of 1/3 cubit length	
2.116	gi-2/3-kuš ₃	Reed of 2/3 cubit length	
(After LANDSBERGER (1959), pp. 191–192)			

Later in the curricular sequence the names of some metrological units crop up in the more advanced list Proto-Ea, whose function was to list different Sumerian readings of single signs. Because the list is ordered by the shapes of the signs, the signs with metrological significance are scattered randomly throughout the 994 entries, amongst sequences dealing with the Sumerian values taken by individual cuneiform signs and their compounds. For instance the metrological unit ninda 'rod' is just one possible reading of the sign GAR:

208	ni-im ₃	niĝ ₂	The sign GAR can be read as nig
209	ĝa₂-ar	ĝar	The sign GAR can be read as gar
210	in-da	ninda	The sign GAR can be read as ninda
211	šu-ku	šuku	The sign sequence U GAR can be read as šuku
212	pa-ad	pad	The sign sequence U GAR can be read as pad
213	ku-ru-um-ma	kurum ₆	The sign sequence U GAR can be read as kurum.
(After CIVIL et al. (1979), p. 40)			

Metrological units written with more than one sign, such as ma-na "mina" and šu-si "finger" make no appearance in Proto-Ea, and nor do measures such as ban_2 , barig, and $eše_3$, whose units are implicit in the writing of the numerical values. The metrological readings of signs in Proto-Ea are as follows:

084	si-la	sila ₃	capacity measure, c.1 litre
114	bu-ru	bur ₃	area measure, c.6.5 ha
210	in-da	ninda	length measure, c.6 m
231	ku-uš	kuš3	length measure, c.0.5 m
345	gu-ur	gur	capacity measure, c.300 litres
365	ša-ar	šar ₂	area measure, c.3600 m^2
689	še-e	še	multi-purpose small unit, 1/180 shekel

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718gi-im3gin2multi-purpose small unit, 1/60 of a sila3, ma-na, or sar806ku-rugur7capacity measure, 3600 gur, c.1,080,000 litres(After CIVIL et al. (1979), pp. 30–63 passim)

Immediately after Proto-Ea, it appears, the students of House F moved on to learning the standard metrological series. They had thus already acquired some systematic knowledge of measures in context (in sub-sequences of the thematic noun list) and learned the contextualised readings of many of the signs for metrological units before they encountered the system as a whole.

2.3 The standard metrological series

Very little has been studied of the Old Babylonian metrological lists since Neugebauer and Sachs established the organisation of the four systems — length, area and volume, weight, and capacity.²⁶ Indeed, as sources for the metrological history of Babylonia they yield nothing once the approximate sizes of the basic units and the relationships between them are known. However, when viewed as the products of scribal education they are potentially interesting once more.

The standard metrological series comprises sections on capacity measure, weights, areas (or volumes), and length in the following ranges:

Capacity:	1/3 sila ₃ – 1 00 00 gur	$(5 \times 60^4 \text{ sila}_3)$	c.0.3 – 65 million litres	
Weight	1/2 še – 1 00 gun	$(3 \times 60^4 \text{ se})$	c.0.05 g – 1,800 kg	
Area:	$1/3 \text{ sar} - 2 \ 00 \ 00 \ \text{bur}_3$	(60^4 sar)	$c.12 m^2 - 47,000 ha$	
Length:	1 šu-si – 1 00 danna	$(3 \times 60^4 \text{ šu-si})$	c.17 mm – 650 km	
(After FRIBERG (1987–90), p. 543)				

		A and the
10 danna 5	10 danna = 5 00 00 (ninda)	
11 danna 6 30	$11 \text{ danna} = 5^{!} 30 00 \text{ (ninda)}$	K
12 danna 6	12 danna = 6 00 00 (ninda)	K
13 danna 6 30	13 danna = 6 30 00 (ninda)	
14 danna 7	$14 \text{ danna} = 7\ 00\ 00\ (\text{ninda})$	VIEW H
16 danna 7 30	$15^{!}$ danna = 7 30 00 (ninda)	VIE A
17 danna 8	$16^{!}$ danna = 8 00 00 (ninda)	
18 danna 8 30	$17^{!}$ danna = 8 30 00 (ninda)	
19 danna 9	$18^{!}$ danna = 9 00 00 (ninda)	
19 danna 9 30	19 danna = 9 30 00 (ninda)	
20 danna 10	20 danna = 10 00 00 (ninda)	
30 danna 15	30 danna = 15 00 00 (ninda)	Sec all
40 danna 20	40 danna = 20 00 00 (ninda)	
50 danna 25	50 danna = 25 00 00 (ninda)	
1 danna 30	1 00 danna = 30 00 00 (ninda)	
		1 - South A

Figure 5: 3N-T 316 = A 30211 (unpublished). Detail of reverse, showing large length measures (1 ninda = 6m; 1 danna = 1800 ninda = 10.8 km).

²⁶ Exactly the same units were used for areas and volumes, volume units being defined as 1 (horizontal) area unit \times 1 cubit height (NEUGEBAUER and SACHS (1945), pp. 4–6).

Extracts from the series could be written in the form of lists — with each entry containing the standard notation for the measures only — or as tables — where the standard writings were supplemented with their sexagesimal equivalents.²⁷ For instance, the reverse of 3N-T 316 contains an extract from the end of the metrological table of lengths (Figure 5).

	Lists	Tables	Total
Capacity measures	3	2	5
Weights			
Areas - volumes			
Lengths		1	1
Total	3	3	6

Figure 6: Metrological extracts on tablets from House F.

Fifteen tablets with extracts from the standard metrological series survive from House F. Some or all of their contents, tablet type, and compositional format can be determined for twelve of them so far (from catalogue records and personal inspection). Almost all identifiable pieces are Type II/2 tablets. On their obverses are a reciprocal table, sections of Proto-Diri, model contracts, and Sumerian proverbs: metrology thus preceded these topics in the House F curriculum. One Type II/1 table of weights has an extract from Proto-Izi on the reverse: metrology thus followed this composition in the House F curriculum. The other fragments appear at this stage of research to have come from Type I or Type II tablets; there is no metrology surviving on tablet types III or IV. Of the six tablets whose contents and compositional format are identifiable, all but one are from the start of the sequence, but there is an even split between tabular and list format.²⁸



Figure 7: 3N-T 594 = IM 58573. The obverse of the Type II tablet (left) shows a teacher's copy of the list of reciprocals, with the student's copy to the right erased. The reverse (right) is an extract from the standard metrological list, with capacity measures from 12 to 19 gur and 3000 to 360,000 gur.

²⁷ FRIBERG (1987–1990), pp. 542–543.

²⁸ Compare the six metrological tablets from the gala-mahs' house (\$1.3): all are tables on fragments of Type I tablets. Three tabulate capacities only, one tabulates both capacities and weights, while two tabulate weights alone. It appears as though a single student had worked his way through the metrological series from the beginning to about the half-way point (TANRET (2002), pp. 100–112).

In short, on present evidence little can be said about metrology within House F, except that its position in the curriculum can be established, and that Type II/2 extracts from the beginning of the compositional sequence apparently predominate the meagre extant record. But it is impossible to determine whether the list and tabular formats had distinct pedagogical functions; neither is there much to be deduced from comparative material (primarily because it is all under-published). However, we can do a great deal more with the much more abundant remains from the standard arithmetical series which immediately followed it in the House F curriculum (§3). First, though, we will jump ahead to the end of the elementary curriculum to examine the use of metrology in model contracts.

2.4 Metrology in use: model contracts

Towards the end of elementary education in House F students were introduced to whole sentences in Sumerian for the first time, in the form of model legal contracts. The genre as a whole, although apparently a relatively common element in scribal schooling, has not yet been studied in depth.²⁹ The contracts from House F concern grain and silver loans, inheritance divisions, and sales of slaves and houses. All of them use metrological units in quasi-realistic contexts, as the following two examples show:

1 (gur) še-gur	300 litres of grain						
maš ₂ 1 gur 1 (barig) 4 ban ₂ še-ta-am ₃	The interest is 100 litres of grain for every 300 litres						
si-ge ₄ -de ₃	To be removed?						
ki lugal-ezen-ta	From Lugal-ezen						
^m a-pil ₂ -ku-ga-X []	to Apil-kuga-[]						
iti sig ₄ -a []	Brick-making month []						
(3N-1.914.x = A.33446, unpublished)							
1/3 sar e ₂ -du ₃ -a da e ₂ diĝir-ga-mi-il	12 m ² built-up house, next to Dingir-gamil's house						
[2]5 sar a-ša ₃ du ₆ a-hu-ni uš-a-du diğir-ga-mil	900 m ² field, the ruin mound of Ahuni, bordering Dingir-gamil's (land)						
[1]2 1/2 sar ĝiš-kiri ₆ <a>-ša₃ id₂ «lugal» lugal zag ĝiš-kiri₆ diĝir-ga-mil	450 m ² date orchard of the royal waterway field next to Dingir-gamil's date orchard						
[1 ĝiš-banšur-zag]-gu-la 1 ĝiš-ga-nu-um-kaš	[1] large [offering table], 1 wooden pot stand for beer,						
[3 ĝiš-dilim ₂] u ₃ niĝ ₂ -gu ₂ -[un]-a igi-4-ĝal ₂ -bi	[3 wooden spoons] and a quarter of its (i.e., the estate's) equipment						
ha-la-ba a-pil ₂ -i ₃ -li ₃ -šu	Apil-ilishu's share.						
(3N-T 342 = IM 58436, lines 10'-16', unpublished)							

In short, some aspects of metrology ran right through the elementary curriculum in House F. However, the focus appears to have been on memorisation and contextual use; there is no evidence that the House F students practised metrological conversions or calculations of any kind (see §4.3).

²⁹ For examples of edited model legal documents see CIVIL (1975), pp. 129–130; WILCKE (1987), pp. 104–107; ROTH (1995), pp. 46–54; VELDHUIS (2000), p. 386, and BODINE (2001).

3. Arithmetic

3.1 The standard arithmetical series

The standard list of multiplications was described long ago by Neugebauer and Sachs, and is very well known.³⁰ Nevertheless, it is useful to summarise its salient features from an educational standpoint (§3.1). Systematic differences in content and textual format across tablet types reflect their pedagogical function (§3.2), while regular omissions from the standard list suggest one or two idiosyncrasies particular to House F (§3.3).

The series starts with a list of one- and two-place reciprocal pairs, encompassing all the regular integers from 2 to 81. It is followed by multiplication 'tables' for sexagesimally regular head numbers from 50 down to 1 15 (see Figure 11), with multiplicands 1–20, 30, 40, and $50.^{31}$ Some series also include the squares and inverse squares of each head number. Neugebauer reconstructed the standard sequence on the basis of what he called 'combined multiplication tables' — or in curricular terminology long extracts on tablet types I and II/2. His 'single multiplication tables' turn out to be tablet types II/1 and III.



Figure 8: 3N-T 261 = UM 55-21-289 (obverse) a verbose Type III multiplication table for 1;40 (left), and 3N-T 608 = UM 55-21-360 (obverse), a terse Type III multiplication table for 3 (right).

Neugebauer also identified three main textual formats for multiplications, and four less common variants.³² We could call Neugebauer's Types A and A' *verbose* formats, in that they repeat the word a-ra₂ 'times' in every line of each table (*h* a-ra₂ 1 *h*, a-ra₂ *m hm* "*h* times 1 is *h*, times *m* is *hm*"). His Types B, B', B'', C, and C', however, are all *terse*, as a-ra₂ 'times' makes at most one appearance in the first line;

³⁰ NEUGEBAUER (1935–1937), I, pp. 32–67 and NEUGEBAUER and SACHS (1945), pp. 19–33.

³¹ In the following paragraph, I abbreviate 'head number' as h and 'multiplicand' as m. I have put the word 'tables' in inverted commas because these tablets are not laid out as formal tables with columnar divisions but as lists like the bulk of the rest of elementary school subject matter (see ROBSON (2003)).

³² NEUGEBAUER and SACHS (1945), p. 20.

thereafter the text is entirely numerical (*m* hm). In fact it turns out that the formats so far attested in House F are all either Type A or Type C; for that reason they will be referred to simply as Verbose and Terse formats, to prevent the confusing proliferation of Types in the discussion (Figure 8). Analogously, the reciprocal tables at the head of the series may be in Verbose format (igi-*n*-g̃al₂-bi 1/n "Its *n*th part is 1/n", e.g. Figure 7) or Terse (n 1/n).³³

Of the 97 House F tablets currently known to contain extracts from the standard multiplication sequence, 32 can be identified as Type III, 38 as Type II, and 10 as Type I (Figure 9). Nine fragments may be from Type I or Type II tablets and the typology of the remaining seven is unknown. Eleven of the twenty-five probable Type II/2 tablets have identifiable compositions on their obverses: eight are tables from towards the end of the multiplication series (Figure 11), while there is one model contract, one sequence of Sumerian proverbs, and one composition yet to be distinguished. There are twenty-one multiplication tables on Type II/1 tablets; apart from the eight multiplication reverses just mentioned, one exemplar each of the thematic noun list (division four), Proto-Lu, Proto-Izi, and a metrological list (Figure 7) have been identified.

Tablet type		Number of	tablet pieces	Percentag	ge of total
Ι		10		10.4	
II		38		39.6	
	II/1 & 2		8		8.3
	II/1 only		13		13.6
	II/2 only		17		17.7
III	•	32		33.3	
I or II		9		9.4	
Unknown		7		7.3	
Total		96		100	

Figure 9: Typology of the tablets bearing multiplication tables in House F.

3.2 Tablet functions and textual formats

Why were three different types of tablet used to record the multiplication series in House F? Looking first at the 34 tablets which bear just one identifiable multiplication or reciprocal table each, namely Types II/1 and III (Figure 10), attested tables are scattered apparently randomly through the series: there are 9 tablets from the first quarter, 8 from the second, 9 from the third, and 7 from the last, and there is little difference between the two tablet types. The picture that emerges from the tablets containing longer extracts from the series is very different, however. On both the Type II/2 (Figure 11) and the Type I tablets (Figure 12) the attested tables are predominantly from the first quarter of the series, namely 18 of the 25 Type II/2 tablets and 6 of the 10 Type Is (69 percent in total). All but one of the remainder are from the second quarter, where it appears that there was a formal section break between the tables for 20 and 18.

This distribution is not peculiar to House F. A simple analysis of the Old Babylonian 'combined multiplication tables' published by Neugebauer before

³³ Cf. NEUGEBAUER and SACHS (1945), p. 12.

Head no.	Tablet Type II/1	Tablet Type III	Total
Reciprocals	1	2	3
50	1		1
48			
45	1		1
44 26 40			
40		1	1
36			
30			
25	3	1	4
24			
22 30			
20			
18		1	1
16 40		1	1
16	1	1	2
15			
12 30	1	2	3
12			
10			
9		1	1
8 20			
8			
7 30	1		1
7 12			
7			
6 40	1		1
6		1	1
5			
4 30	3	1	4
4		1	1
3 45		1	1
3 20			
3	1		1
2 30	1		1
2.24	-		•
215			
2		1	1
1 40		1	1
1 30	1	•	1
1 20	1		1
1 1 5	1	1	1
Total	17	17	31

Figure 10: Tables attested on Type II/1 and Type III tablets from House F.³⁴

 $^{^{34}\,}$ Dotted lines in this and following figures are reading aids; they do not mark formal divisions in the series.

											74										~		ч		
/·ot	912.g	z.60	912.ee	912.11	309.r	912.cc	915.q	912.ff	912.i+	910.1	919.4	912.r	914.p	1111	910.y	912.n	913.h	910.0	912.ii	912.h	912.w	913.t	912.h	913.i	212
Head 1 Tablet	N-T-N8	3N-TS	3N-T	3N-T	3N-T	3N-T	3N-T	3N-T	3N-T	3N-T	3N-T	3N-T	3N-T	3N-T	3N-T	3N-T	3N-T	3N-T	3N-T	3N-T	3N-T	3N-T	3N-T	3N-T	3N-T
Recips	••	•	•	(•)	•	•	•	•	•	•															
50		•	•	•	•	•	•	•	•	•															
48				-	-	-	-	-	-	-															
45				•	•	•	•	•	•	•															
44 26 40							-	-	-	-															
40							•	•	•	•	•	:													
36										(•)			÷	÷				•							
30									•	•	•				(•)										
25																									1
24																									
22 50																									
18																			•						
16 40																			•	(•)					
16																			•	(•)	•	•			
15																			•	(•)	•	•	•		
12 30																			•	•	•		•		
12																					•		•		
10																			1					•	
9																								•	
8 20																									.
8													٦												
7 30						Key		•																	
7 12	1					· ·	=	Attes	sted																
7						-	=	Not a	atteste	ed 	alla at	tected													
6 40						(•)	=	(tabl	ably C	naged	or br	oken)													
5							-	Tabl	et bro	hageo ken	01 01	oken)													
1 30								Tuor	01 010																
4 30																									
3 45																									1
3 20																			† 						1
3																			1						
2 30																			1						
2 24																									
2 15																									
2																									1
1 40																									
1 30																									•
1 20																									•
1 15																									· ·
													acts												
								ist					ontra												
		ast	ast	ast	ast	4	¥	earl		ast	ing	×	el c		8	30		ast	erb	ast		ast	ast	20	sing
λq	50	10 Cž	10 C	10 Cî	10 Cî	lanl	olan	nch	¢ 25	10 C	niss	blan	pou	x 25	× 4	x 4:	x 25	no c	prov	no c	x ?	no c	no c	× 1:	miss
	×	F	-	-	-		~	-	^	-	-		-	~				-				-	_		:

Figure 11: Multiplication and division tables attested on Type II/2 fragments in House F.

_	.t	.k	ij	h.	>.	N	qq	q.	5	
ло.	606	912	912	910	912	912	60	913	39.	912
ad 1 olet	E-	Ľ.	Ę	Ę.	Ĺ.	Ę	Τ9	É.	T-J	Ę
Hea	3N	3N.	3N-	3N	3N	3N	Ż	3N	3N	3N
Recips	•		•	(•)			<u>(</u> ,			
50	•	•	•	(•)						
48	_	(-)	_	(_)						
45	•	(•)	•	•́	•	•				
44 26 40	_	(-)		_	_	_				
40	•	•		•	•	•				
36				(•)	•	•				
30				(•)						
25				(•)						
24				(•)						
22 30				(•)						
20			?	(•)						
18				•			•	•	•	
16 40				•			•	•	•	•
16				•			•	(•)	•	•
15				•••				(•)	•	•
12 30								(•)	•	•
12								(•)	•	(•)
0								(•)	•	•
8 20								•	•	•
8									•	
7 30									•	
7 12										
7										
6 40										
6										
5										
4 30										
4										
3 45										
3 20										
3										
2 30										
2 24										
$\frac{213}{2}$										
$\frac{1}{1}40$										
1 30										
1 20										
1 15										

Figure 12: Multiplication and reciprocal tables attested on Type I tablets in House F.

Tablet Type	I and II/2	('combined' tables)	II/1 and III	('single' tables)
Start of sequence	House F	Neugebauer	House F	Neugebauer
First quarter	24	51	9	56
Second quarter	10	6	8	44
Third quarter		8	9	34
Fourth quarter	1	5	7	25
Total	35	70	34	159

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Figure 13: Distribution of tablet types across the standard series of multiplications.

House F was excavated reveals a striking similarity.³⁵ Of the 70 tablets he listed, 51 of them (72 percent) apparently begin their sequences of multiplications in the first quarter of the series, 6 in the second quarter, 8 in the third, and 5 in the last (Figure 13).³⁶ On the other hand, the number of Neugebauer's 159 'single multiplication tables'³⁷ decreases more or less linearly across the series: 56 are from the first quarter, 44 from the second, 34 from the third, and 25 from the last. While this pattern of attestation does not exactly match the even distribution of tablet types II/1 and III in House F, it is clearly distinct from the heavy skew towards the beginning of the series found in the 'combined multiplication tables' (Tablet Types I and II/2) from House F and elsewhere.³⁸

Neugebauer highlighted the strong correlation between tablet type and textual format:³⁹ some 80–90 percent of his 'combined' multiplication tables (depending on how one defines and counts the tables) are in terse formats and the remainder are verbose. Conversely, about 70–80 percent of the 'single' multiplication tables are verbose and the rest terse. Once again we find similar results in the House F corpus, where formats can be identified: 31 of the 35 Type I and Type II/2 tablets (89 percent), bear tersely formatted tables, while 29 out of the 34 Type II/1 and III tablets (85 percent), are verbose.

In sum, there are two clearly marked distinctions between the 'single' multiplication tables on the one hand and the 'combined' tables on the other. On the one hand, the single tables (on tablet Types II/1 and III) are evenly distributed across the whole series (but with some skew towards the beginning in Neugebauer's sample) and are predominantly verbosely written, while the longer extracts containing sequences of tables are very heavily weighted towards the start of the series and are generally terse. One can also make a further differentiation: it is generally true that the 'single' tables are written in a careful, calligraphic hand with clear line spacing, while the long extracts comprising many tables appear to have been written with little regard for visual appearance: there are generally no line rulings, for instance, and even the columnar divisions are often difficult to make out.

³⁵ NEUGEBAUER (1935–1937), I, pp. 35; II, p. 37; NEUGEBAUER and SACHS (1945), pp. 25–33.

³⁶ However, it is difficult to judge from the descriptions given by NEUGEBAUER and SACHS (1945), pp. 25–33 whether the tablets are fragments or not, and therefore whether complete sequences are attested on them.

³⁷ NEUGEBAUER (1935–1937), I, p. 34; II, p. 36; NEUGEBAUER and SACHS (1945), pp. 20–23.

³⁸ There is little comparative data from known archaeological contexts (\$1.3). The two Type III multiplication tables from Sîn-kāšid's palace in Uruk are for 45 and 22 30 (both terse). The five from the Uruk *Scherbenloch* are for 45, 22 30, 9, 8;20, and 3 (all verbose).

³⁹ NEUGEBAUER (1935–37), I, pp. 62–64.

These three factors combine to suggest a clear pedagogical distinction between the well written, fully worded single tables on the one hand and the hastily scribbled, terse sequences of tables on the other. We have already reviewed Veldhuis's hypothesis ($\S2.1$) that Type II tablets had a dual function: on the obverse (II/1) the student repeatedly copied the teacher's model of an extract (or table) that he was learning for the first time, and then on the reverse (II/2) wrote out a much longer extract from earlier in that same composition, or from one he had already mastered. The evidence from the standard series of multiplication tables presented here not only allows us to confirm that hypothesis but also to draw some further conclusions. First, it appears that Type III tablets were also used in the initial stages of learning an extract, presumably after the student had memorised it well enough to no longer need a model to copy in the Type II/1 pattern. Equally, the Type I tablets appear to have served a similar revision purpose to the Type II/2 tablets, on which students reviewed long stretches of material they were no longer actively working on, or perhaps fitting their most recent achievements into their place in the compositional sequence. Second, and perhaps more interestingly, it seems that while students were given initial exposure to the whole of a composition, by means of short extracts on tablet Types II/1 and III, their revision of that work was much less systematic, starting from the beginning again each time and rarely reaching the end.

This distribution of tablet types across the series is found in other elementary educational compositions too. It is comparable, for instance, to the survival patterns of Old Babylonian tablets from Nippur containing extracts from division one of the thematic noun list, the trees and wooden objects (\$2.1–2). Counting the number of sources for each tablet type over the 707-line composition in Veldhuis's edition,⁴⁰ the following pattern emerges (Figure 14): there are a mean of 44 sources for each of the first five lines, 14 for lines 101–5, four for lines 301–5, three for lines 501–5, and just two for lines 701-5. Dividing the tablet types into their functions of 'first exposure' (Types II/1, III, and probably IV; cf. §2.1) and 'revision' (Types I, II/2, and P), we see that there are never more than four 'first exposures' for any one of the lines sampled but more often one or none. Conversely, the 'revision' tablets are very heavily weighted indeed towards the beginning of the composition (taking into account the commonly occurring damage to the corners of tablets which has lowered the number of attestations for the very first two or three lines). In other words, this suggests that although elementary students in Nippur tended to be taught compositions in their entirety, from beginning to end, all revision in the elementary curriculum was slanted towards the opening sections of compositions to the detriment of their middles and closing lines.

⁴⁰ VELDHUIS (1997), pp. 191–252.

Tablet type	First	expo.	sure	Rev	ision		Unclear	
Line	II/1	III	IV	Ι	II/2	Р		Total
1	1		1		32			34
2			1		35		1	37
3			1		40		1	42
4	1		1		46		2	49
5	1		1		47		2	50
101	1			2	9			12
102	1			2	13			15
103	1			2	13			15
104	1			2	11			14
105	1	1	2	2	9			15
301	2				4	1		7
302	1				2	1		4
303					1	1		2
304					3	1		4
305					4	1		5
501	1			1				2
502	1		1	1	1			3
503				1	1			2
504	1				3			4
505	1				3			4
701				1	1			2
702				1	1			2
703				1				1
704			1	1	1			2
705				1	1			2
Total	15	1	7	18	272	5	6	329

Figure 14: Distribution of tablets over the OB Nippur rescension of the List of trees and wooden objects.

3.3 Missing head numbers

Returning to the standard series of multiplications as attested in House F, nine of the forty known head numbers—namely 48, 44 26 40, 20, 7 12, 7, 5, 3 20, 2 24, and 2 15—do not survive on known tablets. Should we attribute these omissions to the accidents of recovery or to deliberate exclusion from the series? The patterns of attestation make it easier to make definitive statements about the higher head numbers than the lower. The head number 48, for instance, is included in just five of the 71 'combined' tables catalogued by Neugebauer (and just two of those five are from Nippur), compared to twenty-three certain omissions. He lists no 'single' tables for 48. Similarly, 2 15 occurs in two out of nine possible 'combined' tables, neither of them from Nippur, and in no 'singles'. It is not surprising, therefore, that the 48 and 2 15 times tables were apparently not taught in House F. The exclusion of 44 26 40, is rather more surprising: given its place near the start of the standard series it is presumably not simply missing by archaeological accident. On the other hand none of Neugebauer's 'combined' tables appear to omit it, while he lists three 'single' tables for 44 26 40. This is a deliberate but idiosyncratic omission then,

particular to House F—though perhaps a judicious one; none of the other head numbers are three sexagesimal places long. It is probably best to reserve judgement on the remaining six 'missing' head numbers.

4. Beyond elementary education

4.1 The Sumerian literary curriculum in House F

Although, as we have seen (§1.2), the vast majority of mathematical tablets in House F can be assigned to the elementary curriculum on grounds of content and tablet typology, there are four which cannot be. Three of those tablets bear calculations, while the fourth contains an extra-curricular table. Although the table is difficult to place pedagogically (§4.5), it is possible to position the calculations within the 'advanced' curriculum (§4.3), which in House F was dominated by Sumerian literature (§4.2), and to compare this situation with calculations in other school corpora (§4.4). First, however, we need to review what is known of the Sumerian literary curriculum in House F.

Over eighty different literary works have survived from the House, attested on around 600 different tablets. Although we do not have a clear-cut tablet typology from which to deduce a well defined and ordered curriculum, it is possible to at least outline the contents of that curriculum, based on contemporaneous literary catalogues and some basic quantitative methods.⁴¹ First, by simply counting the number of (joined) sources for each composition, it becomes clear that there is one 'mainstream' group of twenty-four literary works, each with a mean of 18 sources, compared to the rest which have on average just 3 attestations. Second, ten of those twenty-four 'mainstream' works comprise a widely-attested curricular grouping that Steve Tinney has labelled the Decad.⁴² The incipits of the Decad members comprise the first ten entries (in the same order) of three Old Babylonian literary catalogues, and have a strong presence in four others. The remaining members of that mainstream grouping, which I have called the House F Fourteen,⁴³ appear on three of those same catalogues, in a fixed order though not clustered together in a single block like the Decad (Figure 15).⁴⁴

The remaining House F literature can be roughly categorised into four groups:⁴⁵ myths, epics and laments (13 works), hymns to kings and deities (11), school narratives, debates and dialogues (7), and literary letters and related short pieces (25). There is also at least one fragment of extracts from a law code⁴⁶ and one tablet containing a fragment of a Gilgamesh myth in Akkadian.⁴⁷

⁴¹ TINNEY (1999); ROBSON (forthcoming).

⁴² TINNEY (1999), pp. 168–170.

⁴³ ROBSON (forthcoming).

⁴⁴ Outside House F, the Decad members are found on an average of 41 Nippur tablets each and 35 non-Nippur tablets. For each of the Fourteen there are, on average, 30 Nippur tablets (outside House F) and 10 from beyond Nippur.

⁴⁵ ROBSON (forthcoming).

⁴⁶ ROTH (1995), p. 250.

⁴⁷ CAVIGNEAUX and RENGER (2000).

	Literary	ETCSL	No. of	Line number of catalogue ⁴⁸						
	composition	no.	sources							
40				N2	L	<i>S1</i>	U1	U2	B4	Y2
D0149	Shulgi Hymn A	2.4.2.01	17	01	[01]	01		04	07	01
D02	Lipit-Eshtar Hymn A	2.5.5.1	12	02	[02]	02	—	05	08	02
D03	The Song of the Hoe	5.5.4	24	03	[03]	04	—	—	09	03
D04	Inana Hymn B	4.07.2	36	04	[04]	03	_	08	03	04
D05	Enlil Hymn A	4.05.1	24	05	05	05		16	10	
D06	Kesh Temple	4.80.2	22	06	06	06		23		_
	Hvmn									
D07	Enki's Journey	1.1.4	9	07	07	07	—	28	24	—
D08	Inana and Ebih	132	18	08	08	08	10	13	02	
D00	Nungal	4 28 1	19	09	09	09	18	14		
D07	Hvmn A	4.20.1	17	07	0)	0)	10	14		
D10	Gilgamesh and	1815	21	10	10	R3	14	09		
210	Huwawa (A)	1101110		10	10	110		0,		
F01	Debate between	5.3.2	19	17	11	_		15	_	_
	Sheep and									
	Grain									
F02	Cursing of	2.1.5	15	18	12	_		17		
	Agade									
F03	Dumuzid's	1.4.3	20	19	13	R4		26		_
	Dream									
F04	Gilgamesh,	1.8.1.4	15	20	14	_		$29?^{50}$		_
	Enkidu and the									
	Nether World									
F05	Instructions of	5.6.1	18	21	15	—		29?	19	_
	Shuruppag									
F06	Debate between	5.3.1	30	25	16	—	—	18	—	—
	Hoe and Plough									
F07	Shulgi Hymn B	2.4.2.2	17	26	17	—	01		—	—
F08	Exploits of	1.6.2	15		18	—		41	—	_
	Ninurta									
F09	Ur Lament	2.2.2	17	32	26	_		44	_	
F10	Schooldays	5.1.1	18	50	_	—	24?51	33?	—	06?
	(Eduba									
	Composition A)					_				
F11	Eduba	5.1.3	14	51	_	R9	24?	_	—	07?
	Composition C									
F12	Eduba	5.4.1	22	52	—	—	24?	_		08?
-	Dialogue 1					4.2	~~	~~		
F13	Farmer's	5.6.3	21	53	—	10	22	35	22	—
F14	Instructions	510	11	~ .				07		
F14	Eduba	5.1.2	11	54	—			07		—
	Composition B									

No of Line number of catalogue⁴⁸ ETCSL

Figure 15: Mainstream Sumerian literary compositions in House F.⁵²

⁴⁸ N2 (ETCSL 0.2.01) from Nippur; L (0.2.02) from Nippur?; S1 (0.2.18) from Sippar; U1 (0.2.03), U2 (0.2.04) from Ur; B4 (0.2.11), Y2 (0.2.12) unprovenanced. R = reverse.

⁴⁹ D01-10 = Decad; F01-14 = House F Fourteen

⁵⁰ This entry, *ud re-a ud sud-ta re²-a*, could be the incipit of either Gilgamesh, Enkidu and the Nether World or the Instructions of Shurrupag.

⁵¹ This incipit, *dumu e₂-dub-ba-a*, could belong to any one of Eduba A, Eduba C, Eduba F (ETCSL 5.1.a, unpublished), *Eduba* Dialogue 1, or *Eduba* Dialogue 3 (ETCSL 5.4.3).

4.2 Mathematics in the Sumerian literary curriculum

References to mathematical achievement and failure in Sumerian literature have been collected before, usually in a misguided attempt to use literary works as unproblematic sources of historical evidence about 'Sumerian school'.⁵³ However, once we recognize that those literary works were themselves elements of a scribal curriculum, as for instance in House F, it becomes interesting and important to study them for the messages that they conveyed to the students about mathematics and the scribes' relationship to it.

Mathematical and metrological elements appear in some of the humorous narratives and dialogues about school life (the so-called *eduba* texts, named after the Sumerian word for school). Although we can occasionally verify that particular details in the narratives are in some sense 'true' in that they concur with other evidence, they are highly unlikely to have been straightforward documentary accounts: after all, their intended audience, the scribal students, already knew exactly what school was like. The narratives often make use of very broad humour to get their message across (or at least broad humour is the only type that we, with our unsophisticated understanding of Sumerian, can currently understand). It may be that other elements of humour lay in the contrast between school life as depicted and as experienced by the students; in that case those apparently realistic details would have served simply to add elements of verisimilitude to otherwise highly fictionalised accounts.⁵⁴

In the most famous of these works, often known by its modern title 'Schooldays', the teacher of an incompetent scribal student is invited home for dinner and bribery, in an attempt to make him ease up on the hapless child. The father flatters the stern teacher shamelessly, saying:

⁵⁹⁻⁶¹ "My little fellow has opened (wide) his hand, (and) you made wisdom enter there; you showed him all the fine points of the scribal art; you (even) made him see the solutions of mathematical and arithmetical (problems)." (*Eduba* Composition A, after KRAMER (1963), p. 239)⁵⁵

An earlier passage in the narrative, however, makes it clear that the teacher had showed him little except the business end of his cane.

⁵² All literary compositions and ancient catalogues are published in the Electronic Text Corpus of Sumerian Literature (BLACK *et al.* 1998–) and cited according to their ETCSL titles and catalogue numbers.

⁵³ E.g. SJÖBERG (1975); NEMET-NEJAT (1993), pp. 5–10.

⁵⁴ Compare Hogwarts, the boarding school for wizards in training, in the highly popular childrens' novels and film about Harry Potter. No child reader has ever set foot in an institution anything like Hogwarts, yet it is still recognisably a school. Its fascination and attraction lies in the fact the judicious combination of realism, fantasy, and humour with which the stories are constructed — just as in the Sumerian school narratives. This, of course, is where the similarity ends.

⁵⁵ No modern critical edition of this composition has ever been published, although there have been single-line composite texts and translations in the public domain for over half a century. I have not attempted to improve on Kramer's translation, apart from the addition of 'even' in the final line.

In a gentler companion piece, sometimes called 'Scribal Activities', a teacher quizzes a student on what he has learned, some three months before he is due to leave school. The student lists everything he has mastered so far, much of which can be matched quite closely to the evidence from the archaeologically recovered elementary tablets themselves. (This is hardly surprising, as one aim of the composition must have been to encourage identification with, and emulation of, this paragon of learning.) The standard metrological lists (§2.3) are as closely associated with the model contracts here are as they are in the elementary curriculum itself.

²⁷⁻²⁹In the final reckoning, what I know of the scribal art will not be taken away! So now I am master of the meaning of tablets, of mathematics, of budgeting, of the whole scribal art. ...

⁴⁰⁻⁴⁸I desire to start writing tablets (professionally): tablets of 1 gur of barley all the way to 600 gur; tablets of 1 shekel all the way to 20 minas. Also any marriage contracts they may bring; and partnership contracts. I can specify verified weights up to 1 talent, and also deeds for the sale of houses, gardens, slaves, financial guarantees, field hire contracts, palm growing contracts, adoption contracts — all those I can draw up. (*Eduba* Composition D, after VANSTIPHOUT 1997: 592–3 and FRIBERG 1987–90: 543)

A third piece is often known as 'The Dialogue between Girini-isag and Enkimanshum' although it is more of a rumbustious slanging match, in which the advanced student Girini-isag belittles and humiliates his younger colleague Enkimanshum (whose defences are often rather ineffectual):

^{19–27}(*Girini-isag speaks*): "You wrote a tablet, but you cannot grasp its meaning. You wrote a letter, but that is the limit for you! Go to divide a plot, and you are not able to divide the plot; go to apportion a field, and you cannot even hold the tape and rod properly; the field pegs you are unable to place; you cannot figure out its shape, so that when wronged men have a quarrel you are not able to bring peace but you allow brother to attack brother. Among the scribes you (alone) are unfit for the clay. What are you fit for? Can anybody tell us?"

^{28–32}(*Enki-manshum replies*): "Why should I be good for nothing? When I go to divide a plot, I can divide it; when I go to apportion a field, I can apportion the pieces, so that when wronged me have a quarrel I soothe their hearts and [...]. Brother will be at peace with brother, their hearts [...]." (Following lines lost) (*Eduba* Dialogue 3, VANSTIPHOUT (1997), p. 589)

Girini-isag's point is that accurate land surveys are needed for legal reasons — inheritance, sales, harvest contracts, for instance. If the surveyor cannot provide his services effectively he will unwittingly cause disputes or prevent them from being settled peacefully.

For the scribal students in House F these three passages helped to define the role of mathematical training within their education. The first extract implies that a truly competent teacher can help even the most hopeless student understand difficult subjects like mathematics. The second outlines what successful students can hope to achieve in the appropriate application of metrological knowledge to legal documents of various kinds, while the last warns of the humiliations of practical incompetence. It is not enough, Girini-isag implies, to have learned your school exercises well if you are physically incapable of putting them into practice.⁵⁶

⁵⁶ *Eduba* composition A is on 18 tablets from House F; it is the tenth member of the House F Fourteen. *Eduba* dialogue 3 is on 3 tablets. No House F sources have yet been identified for *Eduba* composition D but the whole composition is not yet in the public domain.

Two royal praise poems, widely used in the early stages of the Sumerian literary curriculum,⁵⁷ cite mathematical achievement within the repertoire of a good king's accomplishments, bestowed on him by the goddess of scribalism Nisaba-Nanibgal. Their message to the students is that literacy and numeracy are highly desirable skills, valued so much that even kings boast about acquiring them. The following extract from a linguistically elementary hymn to Lipit-Eshtar of Isin (c.1934–24 BC) addresses the king as one who is divinely aided in his literacy and endowed with holy measuring equipment:⁵⁸

¹⁸⁻²⁴Nisaba, the woman radiant with joy, the true woman, the scribe, the lady who knows everything, guides your fingers on the clay: she makes them put beautiful wedges on the tablets and adorns them with a golden stylus. Nisaba generously bestowed upon you the measuring rod, the surveyor's gleaming line, the yardstick, and the tablets which confer wisdom. (Lipit-Eshtar hymn B, BLACK *et al.* (1998–), no. 2.5.5.2)

In this extract, by contrast, the praise singer speaks in the voice of king Shulgi of Ur (c.2094–47 BC), describing his provess in school subjects:⁵⁹

¹²⁻²⁰I, Shulgi the noble, have been blessed with a favourable destiny right from the womb. When I was small, I was at the academy, where I learned the scribal art from the tablets of Sumer and Akkad. None of the nobles could write on clay as I could. There where people regularly went for tutelage in the scribal art, I qualified fully in subtraction, addition, reckoning and accounting. The fair Nanibgal, Nisaba, provided me amply with knowledge and comprehension. I am an experienced scribe who does not neglect a thing. (Shulgi hymn B, BLACK *et al.* (1998–), no. 2.4.2.02)

A praise poem in the voice of king Ishme-Dagan of Isin (c.1953–1935 BC) even self-referentially describes how his varied mathematical and scribal accomplishments have been set to song: 60

^{359-366, 375-377}That the scribal art, in the place of skilled craftsmanship, power; that I have solved calculation problems, counting and reckoning in all their depth and breadth, checking, coefficients, establishing the surface of a field, and laying out the reed measuring-pole; that I have on the podium, my chosen place; that I have learnt with my talented hands, my pure hands, to write the tablets of Sumer and Akkad; that I have lent lustre to the academy by completely mastering the reed stylus and the scribal art; [...] – all these things the scholars and the composers of my songs have put in my great songs and have declared in my hymns. (Ishme-Dagan hymn A+V, BLACK *et al.* (1998–), no. 2.5.4.01)

A few of the Sumerian literary works use metrological concepts as an essential part of their narrative framework. For instance, a 33-line fictionalised letter⁶¹ from Ishbi-Erra (first king of the Isin dynasty, c.2017–1985 BC) to Ibbi-Suen, last king of Ur

⁵⁷ VANSTIPHOUT (1979); TINNEY (1999), pp. 162–168.

⁵⁸ Attested on 3 tablets from House F (and on tablets from other sources).

⁵⁹ It is the seventh member of the House F Fourteen (and widely attested elsewhere).

⁶⁰ Attested on 3 tablets from House F (and on tablets from other sources).

⁶¹ HUBER (2001) has shown convincingly that, on the grounds of grammatical, stylistic, and historical anachronisms the Royal Correspondence of Ur cannot be considered to be 'authentic'. If it ever had any 'historical core' it has been almost completely lost in fictional overlay and pedagogically-motivated accretions.

(c.2028–2004 BC), describes how he, while still in the latter king's service, has been sent north to buy grain in order to alleviate the famine in the south, but is held back by incursions of nomadic Martu people: 62

¹⁻²Say to Ibbi-Suen, my lord: this is what Ishbi-Erra, your servant, says:

 $^{3-6}$ You ordered me to travel to Isin and Kazallu to purchase grain. With grain reaching the exchange rate of 1 shekel of silver per gur, 20 talents of silver have been invested for the purchase of grain.

⁷⁻¹²I heard news that the hostile Martu have entered inside your territories. I entered with 72,000 gur of grain — the entire amount of grain — inside Isin. Now I have let the Martu, all of them, penetrate inside the Land, and one by one I have seized all the fortifications therein. Because of the Martu, I am unable to hand over this grain for threshing. They are stronger than me, while I am condemned to sitting around.

¹³⁻¹⁶Let my lord repair 600 barges of 120 gur draught each; 72 solid boats, 20, 30 bows,
[40] rudders (?), 50 and 60 (?) boat doors on the boats (?), may he also all the boats.
... (after BLACK *et al.* (1998–), no. 3.1.17)

The letter reads suspiciously like an OB school mathematics problem: the first paragraph gives the silver-grain exchange rate and the total amount of silver available (72,000 shekels); in the second the silver has been correctly converted into grain. Next that huge capacity measure is divided equally among large boats (cf. the contextualised large capacity measures in the list of trees and wooden objects, §2.2). As is typical for school mathematical problems, the numbers are conspicuously round and easy to calculate with.⁶³ The numbers in the final, damaged part of the section quoted are reminiscent of the final multiplicands of a standard multiplication table (§3.1) or the sexagesimal fractions 1/3, 1/2, [2/3], 5/6.⁶⁴ The letter, at one level, is no more than a pretext to show simple mathematics and metrology at work in a quasi-realistic context.

The longer composition now known as 'The Farmer's Instructions'⁶⁵ uses school mathematics in a very different way. Ostensibly it is a description of the agricultural year from irrigation to harvest, but it is hardly pastoral in tone. Central to its whole rationale are the standard work obligations by which state institutions of the twenty-first century BC measured out agricultural labour to contract managers and their work gangs.⁶⁶ A short extract from the 111-line composition is enough to catch its flavour:

²³⁻²⁹The plough oxen will have back-up oxen. The attachments of ox to ox should be loose. Each plough will have a back-up plough. The assigned task for one plough is 180 iku (c.65

⁶² One attestation from House F; several other sources known.

⁶³ FRIBERG (1987–90), p. 539.

⁶⁴ These numbers all refer to wooden objects that I suspect may turn out to be identifiable from the boats section of the list of trees and wooden objects (VELDHUIS (1997)). Only one known tablet, IM 44134, preserves the composition at this point. It is held in the Iraq Museum and was not available for collation.

⁶⁵ It is the thirteenth member of the House F Fourteen (and well attested elsewhere in Nippur).

⁶⁶ CIVIL (1994), pp. 75–78, ROBSON (1999), pp. 138–166.

ha), but if you build the implement at 144 iku (c.2 ha), the work will be pleasantly performed for you. 180 (?) sila of grain (c.180 litres) will be spent on each 18 iku area (c.6 1/2 ha).

³⁰⁻³⁴After working one plough's area with a *bardil* plough, and after working the *bardil* plough's area with a *tugsig* plough, till it with the *tuggur* plough. Harrow once, twice, three times. When you flatten the stubborn spots with a heavy maul, the handle of your maul should be securely attached, otherwise it will not perform as needed. (BLACK *et al.* (1998), no. 5.6.3)

The Farmer's Instructions is reminiscent of a small group of Sumerian literary compositions recently studied by Niek Veldhuis.⁶⁷ He has highlighted the intimate lexical and pedagogical relationship between the standard list of fish and birds (division four of the thematic noun list, see §2.1) and two works now known as 'Home of the Fish'⁶⁸ and 'Nanshe and the Birds'.⁶⁹ But whereas they provide a literary framework for naming and describing fish and birds, The Farmer's Instructions sets out to sugar the bitter pill of learning agricultural work rates. It was probably several hundred years behind contemporary scribal practice by the time it was taught in House F, but so was much of the other literature taught there (as can be seen from the regnal dates of the kings referred to in the extracts quoted in this section).

4.3 The good, the bad, and the ugly: calculations of reciprocals

By a great stroke of fortune, one tablet has survived from House F that bears both Sumerian literature and a mathematical calculation. They are on the same sort of tablet as the elementary Type III, which was commonly used to write single-column extracts of up to 60 lines of literary works (and for that reason called Type S in this context).⁷⁰ The literary extract is from the first lines of a composition now known as 'The Advice of a Supervisor to a Younger Scribe', one of the curricular grouping discussed above (§4.2) whose fictionalised setting is the school and whose aim is to instil professional identity and pride into trainee scribes:

 $^{1-2}$ (*The supervisor speaks:*) "One-time member of the school, come here to me, and let me explain to you what my teacher revealed.

^{3–8}"Like you, I was once a youth and had a mentor. The teacher assigned a task to me — it was man's work. Like a springing reed, I leapt up and put myself to work. I did not depart from my teacher's instructions, and I did not start doing things on my own initiative. My mentor was delighted with my work on the assignment. He rejoiced that I was humble before him and he spoke in my favour.

 $^{9-15}$ "I just did whatever he outlined for me — everything was always in its place. Only a fool would have deviated from his instructions. He guided my hand on the clay and kept me on the right path. He made me eloquent with words and gave me advice. He focused my eyes on the rules which guide a man with a task: zeal is proper for a task, time-wasting is taboo; anyone who wastes time on his task is neglecting his task.

 $^{16-20}$ He did not vaunt his knowledge: his words were modest. If he had vaunted his knowledge, people would have frowned. Do not waste time, do not rest at night — get on

⁶⁷ VELDHUIS (2001), esp. §3.2.

⁶⁸ BLACK *et al.* (1998–), no. 5.9.1.

⁶⁹ VELDHUIS (2001), BLACK *et al.* (1998–), no. 4.14.3.

⁷⁰ TINNEY (1999), p. 160.

with that work! Do not reject the pleasurable company of a mentor or his assistant: once you have come into contact with such great brains, you will make your own words more worthy.

^{21–22}"And another thing: you will never return to your blinkered vision; that would be greatly to demean due deference, the decency of mankind."

(BLACK *et al.* (1998–), no. 5.1.3. 3N-T 362 + 3N-T 366 = IM 58446 + 58447 (UM cast) obverse 1–19, reverse 1–3. Obverse unpublished; reverse ROBSON (2000), p. 22)



Figure 16: 3N-T 362+366 (rev). (ROBSON (2000), fig. 2).

Below this, the rest of the reverse is taken up with a calculation of regular reciprocal pairs⁷¹ using a method that SACHS (1947) called "The Technique" (Figure 16). Other tablets with similar arrangements of numbers are known, as well as one very damaged tablet of unknown provenance which originally contained twelve worked examples with instructions.⁷² Like much Old Babylonian mathematics, although it first appears to be analogous to modern algebraic operations it can in fact be best understood in terms of very concrete manipulations of lines and areas.⁷³ The best preserved of the twelve problems runs as follows:

What is the reciprocal of 2;[13] 20?⁷⁴ [You, in your] working: Find the reciprocal of 0;03 20. [You will see] 18. Multiply 18 by 2;10. [You will see] 39. Add 1. [You will see] 40. Take the reciprocal of 40. [You will see] 1 30. Multiply 1 30 by 18. You will see 27. Your reciprocal is 27. [That is the method.]

(VAT 6505, II 8–16. NEUGEBAUER (1935–1937), I, pp. 270–273, II pls. 14, 43; SACHS (1947), pp. 226–227)

⁷¹ ROBSON (2000), no. 2.

⁷² For the most recent discussion, see ROBSON (2000), p. 21.

⁷³ HØYRUP (1990) and (2002).

⁷⁴ I have assigned arbitrary absolute sexagesimal value to the numbers in this problem and those in the following discussion.



Figure 17: Finding sexagesimally regular reciprocals using The Technique.

We can plug the numbers from our House F tablet into this solution. The product of any reciprocal pair is, by definition, 1. We can therefore imagine 17;46 40 as the side of a rectangle whose area is 1 (Figure 17(a)); the task is to find the length of the other side. We can measure off a part of the first side, so that it has a length that is in the standard reciprocal table — in this case 0.0640, whose reciprocal is 9. We can thus draw another rectangle with lines of these lengths, whose area will also be 1 (Figure 17(b)). This gives us an L-shaped figure. We can fill it in to make a rectangle by multiplying the 9 by 17;40, the part of the original length that we haven't used yet - 2 39 (Figure 17(c)). Add 1, the area of the 9 by 0;06 40 rectangle. The total area is 2 40. This new large rectangle, 9 by 17;46 40, is 2 40 times bigger than our original rectangle, with area 1. Therefore 9 is 2 40 times bigger than our mystery reciprocal. We divide 9 by 2 40 by finding the inverse of 2 40 — 0;00 22 30 — and multiplying. The reciprocal we wanted to find is thus $9 \times 0;00\ 22\ 30 = 0;03\ 22\ 30$ (Figure 17(d)). This is the number in the middle of the calculation. The scribe then checks his result by working backwards from 0:03 22 30 to 17:46 40 again.

The other two calculations identified so far on House F tablets are also attempts to find reciprocals, but conspicuously less successful than the first. The longest, written on the back of a roughly made, approximately square tablet, reads:



Figure 18: 3N-T 611 = A 30279 (unpublished), reverse. A student's calculation.

Nothing remains on the obverse apart from a few apparently random signs. The first part of the calculation is a squaring of the number 16;40, set out in the usual way with the multiplicands and product aligned vertically,⁷⁵ albeit with an unexplained extra copy of the 16;40.⁷⁶ The answer is correctly given as 4 37;46 40, but the 9 written immediately to the left strongly suggests that The Technique was then used to find its reciprocal.

As in our first example, the student has split 4 37;46 40 into 4 37; 40 and 0;06 40. He has appropriately taken the reciprocal of the latter — 9 — and multiplied it by the former, adding 1 to the result. However, instead of arriving at 41 39 + 1 = 41 40, our student has lost a sexagesimal place and found 41;39 + 1 = 42;39. Unable to go further with his calculation (for the next stage is to find the reciprocal of the number just found, but his is coprime to 60) he has abandoned the exercise there. The correct answer would have been 0;00 12 57 36.⁷⁷



Figure 19: 3N-T 605 = UM 55-21-357 (obv), ROBSON (2000), no. 1. An attempted reciprocal calculation.

The last calculation of the three (Figure 19) is the most pitiful. Writing on a Type S tablet like the first example, the student has got no further than:

4 26 40	4;26 40
igi-bi 2 13 20	Its reciprocal is 2;13 20

The double ruling underneath shows that he thinks he has finished, although he has done nothing more than halve the first number (Figure 19). The correct result is 0;13 30.

Two of the three numbers whose reciprocals are to be found come from the standard school sequence of reciprocal pairs to which all other known exemplars of

⁷⁵ ROBSON (1999), pp. 250–252.

⁷⁶ Three tablets from the early excavations at Nippur also bear squaring calculations in the same format: CBS 3551 (NEUGEBAUER and SACHS (1945), p. 36), HS 232 (FRIBERG (1983), p. 82), and N 3971 (ROBSON (1999), p. 275).

⁷⁷ I do not yet have any explanation for the 20 and 50 written to the left of the calculation; presumably they relate to intermediate steps in the procedure. Compare similarly positioned auxiliary numbers in calculations from Ur, e.g. *UET* 6/2 387 (ROBSON (1999), p. 249).

this exercise belong.⁷⁸ The sequence is constructed by successively doubling/halving an initial pair 2 05 and 28 48. Our two are eighth (4 26 40) and tenth (17 46 40) respectively. On the other hand, 4 37 46 40 does not, as far as I can ascertain, fit the pattern; presumably it was chosen because, like the other two, it terminates in the string 6 40. One possible interpretation of this commonality is that three students were set similar problems at the same time, using a common method and a common starting point but requiring different numerical solutions. One of three used the method correctly, producing the right answer and checking his results; the second chose the appropriate method but could not apply it satisfactorily, while the third had missed the point of the exercise entirely. We can find corroboration for this hypothesis in groupings of other sorts of calculations from the city of Ur.

4.4 Calculations in other curricula

Some forty-five arithmetical calculations are known to have come from 'No. 1 Broad Street' in Old Babylonian Ur (§1.3), as I have discussed elsewhere.⁷⁹ One of them uses The Technique to find the reciprocal 28 48 of 2 05, that is, the first pair in the standard sequence just discussed.⁸⁰ A further three calculate the squares of sexagesimally regular numbers,⁸¹ the first of which is identical to the number in 3N-T 611, namely 16;40. In all cases the visual layout of the calculations is identical to those on the House F tablets: the calculation proceeds downwards, with reciprocal pairs written in horizontal alignment (but with almost no space separating them) and numbers to be squared written twice in vertical alignment. Products are recorded underneath multiplicands. No rulings, horizontal or vertical, are used (Figure 20).



Figure 20: UET 6/2 295 and 211 (rev.). Calculations from No. 1 Broad Street, Ur. (ROBSON (1999), figs. A.5.6, A.5.7).

The largest group of Broad Street calculations, however, is not paralleled in House F. On some 20 tablets, sequential multiplications are recorded either side of a

⁷⁸ ROBSON (1999), p. 23.

⁷⁹ ROBSON (1999), pp. 246–272. For an alternative interpretation of this material, see FRIBERG (2000).

⁸⁰ UET 6/2 295, ROBSON (1999), p. 250.

⁸¹ UET 6/2 211, 222, and 321, ROBSON (1999), pp. 251–252.

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Figure 21: UET 6/2 236 and 247 (rev). Sequences of multiplications from No.1 Broad Street, Ur. (ROBSON (1999), figs. A.5.10, A.5.14).

vertical line.⁸² Yet just as the three House F reciprocal calculations share a common element in the number 6 40, two subgroups of the Broad Streets multiplications have multiplicands in common. In five exemplars (e.g., Figure 21, left) the third multiplicand is 3 and the fourth 0;06 (i.e., the divisor 10). In another five (and a further two possible damaged specimens) the fourth multiplicand is always 6 40 (e.g., Figure 21, right).

I concluded my study of those tablets from Ur with the following inferences, all but the last of which we can apply to the calculations in House F as well:⁸³

- first, that students were set problems to solve, and that the mathematics education was not restricted to learning arithmetical and metrological tables, and to copying out model solutions;
- second, that [small groups of] students were set problems of the same type but with different values for the variables — perhaps from the 'catalogue'-type lists [that teachers kept of appropriate whole-integer parameters — or from standard sequences such as the halved and doubled reciprocal pairs];
- third, that they were taught to lay out their calculations in standard formats;
- fourth, that students were prone to both calculation (especially place value) errors and mistakes through misreading [or mis-remembering] coefficient lists and arithmetical tables;
- fifth, and most speculatively, that students knew the numerical results they were aiming for, and were not above fudging their calculations to fit.⁸⁴

Nevertheless, however similar the content and organisation of the calculations from the two schools might be, it is clear that they were performed in rather different

⁸² ROBSON (1999), pp. 252–264.

⁸³ It was not possible to identify the exact problems that had been set for the Broad Street students to work on, but it has been attempted in one other instance. YBC 7289, an unprovenanced round tablet bearing a diagram and calculation of the length of the diagonal of a square, can confidently be linked to an entry in the coefficient list YBC 7243 and, more speculatively, to the set of geometrical problems about squares on BM 15285 (FOWLER and ROBSON (1998))

⁸⁴ ROBSON (1999), pp. 263–264; further comments in square brackets.

curricular contexts. Most conspicuously, the mathematical work from Ur is not on square or Type S tablets, but on round tablets identical to the Nippur elementary Type IV (§2.1). Further, while one House F exemplar is found on the same tablet as a Sumerian school narrative, on the obverse of all but four of the Broad Street examples are extracts from Sumerian proverb collections⁸⁵ — which in Nippur belong to the end of the elementary curricular sequence. It would be tempting to ascribe this difference to geographical variation, if it were not for the existence of a group of calculations from Area TB, not 100 metres from House F in Nippur (Figure 1).

House B in Area TB (Level II) yielded some 53 tablets in the season before House F was excavated, all but a handful of which bore school-related subject matter. ⁸⁶ It was much more substantial house than F, with five rooms off a central courtyard. The majority of tablets were found in that courtyard, although there were no schoolyard fittings such as benches or recycling bins as at House F or the *galamahs*' house in Sippir Amnānum (§1.3).⁸⁷ Stone dates the school level to c.1870– 1800, some 60–130 years earlier than House F but roughly contemporary with Broad Street.⁸⁸ At least nine of the school tablets have been catalogued as mathematical, and five have been published; all the legible pieces bear calculations.⁸⁹

Two square shaped tablets (like 3N-T 611 above) carry calculations about squares, but they differ from the examples discussed earlier in that the problem is recorded on them too. In both cases the task is to find the area of a small square — 2/3 cubit 9 fingers long (c.480 mm) and 1/3 cubit 1/2 finger long (c.175 mm) respectively. The question and answer are written on the bottom right corner, and the calculation (without the answer) in sexagesimal place value system on the top left. This format of problem is attested on five other square tablets, at least three of which are also from Nippur.⁹⁰ Maybe they are precursors to, or variants on, the squarings from House F and Broad Street. However, whereas the House B examples all involve the conversion of metrological units to their equivalents in the sexagesimal place value system and back again (as in the standard metrological tables, §2.3), there is no evidence at all for metrological elements in the House F or Broad Street calculations.

Most interesting for our present discussion, though, are the three tablets bearing reciprocal calculations. Like the three from House F, two of the tablets can be

⁸⁵ ALSTER (1997), pp. 306–328. The unprovenanced Type IV tablet YBC 7345 also bears a proverb on the obverse and calculations (sequential multiplications) on the reverse (ALSTER (1997), pl. 130).

⁸⁶ I did not include this house in the inventory of comparative data (§1.3) as almost nothing is published about its tablets. My information comes from matching excavation numbers of published tablets with unpublished excavation records kept at the University Museum, Philadelphia. See also VELDUIS (2000), pp. 387–388.

⁸⁷ STONE (1987), pp. 84–85, pls. 29–30; TANRET (2002), pp. 142–149.

⁸⁸ STONE (1987), p. 119.

⁸⁹ 2N-T 30 (squaring), 2N-T 115 (fragment), 2N-T 116 (squaring): NEUGEBAUER and SACHS (1984); 2N-T 496 (reciprocals): AL-FOUADI (1979), no. 134; 2N-T 500 (reciprocals): GORDON (1959), no. XXX, pl. 70. Further details in ROBSON (2000), p. 19, table 2.

⁹⁰ Listed in ROBSON (1999), p. 12.

classified as Type S, and their textual format and contents are strikingly similar too. 2N-T 500 uses the same reciprocal pair as 3N-T 362+366 (Figure 16), namely 17 46 40 and 3 22 30, and the same layout as 3N-T 605 (Figure 19), with a double ruling underneath the statement of the problem. In this case, though, the student has solved the problem correctly underneath, in a format identical to the House F and Broad Street examples, after an abortive attempt on the other side of the tablet.⁹¹ On 2N-T 496 the reciprocal to be found is the apparently the subject of the squaring on the House F tablet (and one of the Broad Street ones), namely 16 40. All that survives, however, is the answer below — igi-bi 3 36 'Its reciprocal is 3 36' — with no workings shown under the double ruling, so we cannot be sure that the problem was not incorrectly solved (as in 3N-T 605, the only other reciprocal calculation with no workings). The last tablet of the three, 2N-T 115, is an "irregularly shaped fragment" bearing two damaged lines which, if correctly calculated, can be restored as 9 28 5[3 20] / igi-bi 6 1[9 41 15]. Like the House F reciprocal pairs, these numbers all belong to the halved and doubled sequence derived from 2 05 and 28 48 — and all end in 6 40 (in once case 3 20) as well. Is this simply coincidence?

As I have described them so far, the House B reciprocal calculations sound much more like those from House F than the ones from Broad Street. However, like the Broad Street tablets, two of them bear Sumerian proverbs rather than longer literary extracts. On the other hand, both of those proverbs are about failures in scribal behaviour, which situates them rather closer to the school narrative on 3N-T 362+366 than they might otherwise appear:

A foolish scribe: the most backward among his colleagues (2N-T 496, cf. SP 2.42: ALSTER (1997), pp. 53, 304.

A chattering scribe: his guilt is very great (2N-T 500, SP 2.52: ALSTER (1997), p. 55)

Both are from Sumerian Proverb Collection 2+6, which also happens to be the best attested of the Sumerian proverb collections from House F.⁹²

There is an interesting contradiction here: on the one hand, the House B reciprocals are on the same tablet types as House F, use the same introductory layout with statement and double ruling, and use the same reciprocals in 6 40; on the other, they share tablets with Sumerian proverbs like those from Broad Street in Ur (but which are on different a tablet type and do not state the problem). However, the sample at our disposal is undoubtedly too small to confidently assign subject correlation to diachronic change and tablet typology to geographical variation. It is enough for the moment to see that while there are some extraordinary consistencies in both the broad sweep and the detail of calculations taught in schools, there was by no means a 'national' curriculum which all teachers followed.⁹³

⁹¹ ROBSON (2000), pp. 20–21.

⁹² By contrast, of the 45 tablets from Broad Street that bear both proverbs and calculations, only 4 (9 percent) have extracts from the 'scribes' section of Sumerian Proverb Collection 2+6 (ROBSON (1999), p. 246).

⁹³ House F has yielded no mathematical problem texts — that is, documents that set out a mathematical problem to be solved (ROBSON (1999), pp. 7–8). Discounting the Sîn-kāšid school tablets (§1.3) and those in the *gala-mahs'* house (solely elementary exercises, which would not therefore be expected to include mathematical problems), the only archaeologically-defined school corpora containing mathematical problems are from the 'Scholar's House' in Mê-Turān with one tablet of 10 mixed problems (AL-RAWI and ROAF

4.5 An extra-curricular table

Finally, there is just one mathematical tablet from House F which cannot be securely related to other elements of the scribal curriculum (Figure 22):

[1].E 1 $IB_2.SI_8$	[1] is the square of 1	
$[4].E 2 IB_2.SI_8$	[4] is the square of 2	1.0 1. 1.0 1
$[9].E 3 IB_2.SI_8$	[9] is the square of 3	
[16].E 4 IB ₂ .SI ₈	[16] is the square of 4	一 前 师 一 举
[25].E 5 IB ₂ .SI ₈	[25] is the square of 5	
36.e 6 ib ₂ .si ₈	36 is the square of 6	之下 机 战 一 华
49.е 7 ів ₂ .si ₈	49 is the square of 7	金二十十十十十十十十十十十十十十十十十十十十十十十十十十十十十十十十十十十十十
1 04.e 8 ib ₂ .si ₈	1 04 is square of 8	Aline in the state
1 21.e 9 ib ₂ .si ₈	1 21 is the square of 9	以上ティア日本につい
1 40.e 10 ib ₂ .si ₈	1 40 is the square of 10	TA THN
2 01.e 11 ib ₂ .si ₈	2 01 is the square of 11	in all all all all all all all all all al
2 24.e 12 ib ₂ .si ₈	2 24 is the square of 12	HALL STILL
[2 49].°E 13į IB₂.SI8	[2 49] is the square of 13	Jack and Della
[3 16.E 14] IB ₂ .SI ₈	[3 16] is the square of [14]	
[3 45.e 15 ib ₂ .si ₈]	[3 45 is the square of 15]	
[4 16.e 16 ib ₂ .si ₈]	[4 16 is the square of 16]	
		15
[4 49.e 17 ib ₂ .si ₈]	[4 49 is the square of 17]	
[5 24.e 18 ib ₂ .si ₈]	[5 24 is the square of 18]	S SI CONTRACTOR
[6 01.e 19 ib ₂ .si ₈]	[6 01 is the square of 19]	1 Lan maxima
[6 40.e 20] ib ₂ .si ₈	[6 40 is] the square [of 20]	A THE GRADE
[7 21.e 21] ib ₂ .si ₈	[7 21 is] the square [of 21]	
8 04. °E 22į [IB ₂ .SI ₈]	8 04 [is the square of 22]	
[8] 49.E 23 IB ₂ .SI ₈	[8] 49 is the square of 23	
[9 36].e 24 ib ₂ .si ₈	[9] 36 is the square of 24	
[10 2]5.e 25 ib ₂ .si ₈	[10 2]5 is the square of 25	A THE REAL PROPERTY OF
[11 1]6.E 26 IB ₂ .SI ₈	[11 1]6 is the square of 26	A CONTRACTOR OF A CONTRACTOR OFTA CONTRACTOR O
[12 09].E 27 IB ₂ .SI ₈	[12 09] is the square of 27	minte
[13 04].E 28 IB ₂ .SI ₈	[13 04] is the square of 28	
[14 01].e 1 ib ₂ .si ₈	[14 01] is the square of 29	a Bernard
[15].E 1 IB ₂ .SI ₈	[15 00] is the square of 30	

Figure 22: 3N-T 604 = UM 55-21-356 (unpublished). An inverse list of squares from House F.

It bears an inverse list of squares, which it would perhaps be tendentious to connect with the squaring exercise on 3N-T 611 (§4.3). It is a well attested table: Neugebauer and Sachs list eighteen other exemplars,⁹⁴ thirteen of which are in this format; six of those thirteen are also from Nippur.⁹⁵

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⁽¹⁹⁸⁴⁾⁾ and Broad Street, from which probably six tablets contain mathematical problems (CHARPIN (1986), pp. 451–452, 481–482). We might also count the two squaring exercises from House B in Nippur TB (above).

⁹⁴ NEUGEBAUER (1935–1937), I, pp. 70–71; NEUGEBAUER and SACHS (1945), pp. 33–34.

⁹⁵ Three of the four mathematical tablets from No. 7 Quiet Street in Ur are tables like this: 1 table of squares, 1 inverse table of squares, 1 inverse table of cubes (§1.3).

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5. Conclusions

It turns out that a wealth of interesting insights can be gained from mathematical material that has traditionally been dismissed as unimportant and trivial. An awareness of archaeological and social context can illuminate the dullest of texts. However, we should be careful not to blithely generalise the conclusions reached about House F to the whole of Mesopotamia, or even to Babylonia or Nippur: one of the most striking outcomes of this study has been to highlight both the variations large and small between individual corpora of tablets, and the virtual impossibility of ascribing those differences to diachronic change, geographical variation or personal choice (although it appears that this last was more pervasive than we might have thought). To sum up our findings about House F, a small scribal school operating in urban Nippur in the mid-eighteenth century BC:

We can accurately attribute the memorisation of standard metrological and mathematical series to the third phase of elementary education in House F. Metrology was taught before multiplication but it was apparently less important (at least, many fewer tablets survive); it is not yet clear why this is so. Nor is it yet possible to distinguish the didactic roles of metrological lists and tables; they cannot obviously be assigned to 'first exposure' and 'revision' functions. A metrological thread ran right through the curriculum, from ordered lists of metrologically-related objects in the second-phase thematic noun lists, through contextualised metrology in fourth-phase model contracts, to enumerations of metrological constants in the Sumerian literary composition 'The Farmer's Instructions'.

After mastering metrology, the students were probably taught the whole of the multiplication series, in verbose form on Type III and Type II/1 tablets; they revised them frequently, in terse form on tablet Types I and Type II/2. (Type IV tablets were not used for mathematical subjects in House F.) However, it was rare to revise more than the first section or two (breaking between the tables for 20 and 18). 'Tables' is rather a misnomer for this exercise, it turns out: rather, the students were memorising lists of number facts. In fact, the mathematical thrust of the elementary curriculum as a whole can be summarised as the recognition of numbers, weights, and measures in context, and their memorisation in sequence.

Calculations — active mathematics — belonged to the advanced curriculum along with Sumerian literature, some of which had been deliberately written or adapted for specifically mathematical aims, while rather more of it was geared to instilling a sense of professional pride in numeracy and literacy in trainee scribes. The few examples we have, of finding squares and regular reciprocals, might suggest that students found arithmetic difficult and made frequent mistakes. There is a similarity in subject matter and calculation format that extends beyond this single school, to nearby House B and to Broad Street in Ur. At Ur, though, calculations were practised on Type IV tablets, while the students were learning Sumerian proverbs. We do not yet know the order of the curriculum in the Ur school-houses. In House B (contemporary with Broad Street, older than House F), the tablets and mathematical exercises were nearly identical to those in House F but appeared in the same curricular context as Broad Street. House F provides no evidence, direct or indirect, for the use of mathematical problem texts, or for any practice at all in additions and subtractions. As work on the tablets from House F and its neighbours progresses, however, these conclusions will undoubtedly be refined, corrected, and expanded.

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